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ARISTOTLE ON MIXTURES*

QUESTIONS about mixture and combination were among the most central topics discussed by the earliest philosophers. Indeed, Anaxagoras, Anaximines, Empedocles, and others might be described as philosophers and *chemists*. Aristotle, in *De Generatione et Corruptione* (Bk. I, Ch. 10), attempted to solve certain puzzles and to explain mixing. There was some medieval discussion of the topic of mixture, but it has been virtually ignored since then.[1] The topic is worth reviving, both for its

* The present paper comes out of research on mixtures, matter, and mass terms which got its real start during a visit to the University of Auckland in 1973. A second visit for twenty months in 1979 and 1980 provided the opportunity to give a number of lectures on these topics and to finish a great deal of this work.

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[1] For commentaries on Aristotle, and historical discussion of the topic, see Alexander of Aphrodisias, *Peri Kraseōs kai Auxēseōs* ("On Fusion and Augmentation"), in Ivo Bruns ed., *Supplementum Aristotelicum*, vol. 2, part 2, *Scripta Minora* (Berlin: Reimer, 1892) 213-238; Cyril Bailey, *The Greek Atomists and Epicurus* (Oxford, 1928); Ida Freund, *The Study of Chemical Composition: An Account of its Method and Historical Development* (New York: Cambridge, 1904; reprinted New York: Dover, 1968); David J. Furley, *Two Studies in the Greek Atomists*, Study I (Princeton, N.J.: University Press, 1967); H. H. Joachim, *Aristotle* "On Coming-to-be and Passing Away," (Oxford, 1922); G. S. Kirk and J. E. Raven, *The Presocratic Philosophers* (New York: Cambridge, 1957) chap. xv; J. R. Partington, *A History of Chemistry*, vol. I (London: Macmillan, 1970); Rosamond Kent Sprague, "An Anonymous Argument against Mixture," *Mnemosynē*, 26 (1973) 230-233; and Gregory Vlastos, "The Physical Theory of Anaxagoras," *Philosophical Review*, 59 (1950) 31-57.

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own sake as a philosophical problem, and also for its connection to various questions about matter and about mass terms.

1. *The Problem.* Aristotle begins his discussion by asking whether mixture (*mixis*) is even possible, for "as some say": (i) if the ingredients continue to exist in the supposed mixture and are not altered, then they are not really mixed; (ii) if one ingredient is destroyed, then the ingredients have not been mixed, but one ingredient has being and the other does not, whereas what has been mixed should still be what it was before; (iii) if both ingredients are destroyed, then they are not mixed since they do not even still exist (327a29-b7).

It is not clear at all who the "some" are who actually advanced this puzzle, but we can let this question pass and tackle the puzzle itself and Aristotle's solution. First, he gives some examples. (i) In a combination of unmashed beans and rice, the ingredients continue to exist; they are not altered; they are merely physically juxtaposed; we have a mere combination (*synthesis,* 327b35) and not a true mixture.[2] (ii) In burning, wood does not mix with fire, but the fire comes-to-be and the wood passes-away. Properties and states do not mix with *things,* for we see them persisting unaltered (a white man is not a mixture of white and man). (iii) Properties and states cannot be mixed with each other (e.g., knowledge and white), for they cannot themselves exist independently (cf. *Categories* 2, 5).

Aristotle then seems to have certain conditions that a true mixture must satisfy.

Axiom 1. Each of the ingredients in a mixture must have originally existed separately (327b23).

Axiom 2. The ingredients of a mixture can be separated again (327b29).

These two axioms do not tell us what mixtures are, however, for a mere combination like the beans and rice will satisfy them, and Aristotle does not wish to count that as a true mixture. How then could these two axioms hold if it is also required that the ingredients do not persist unaltered? Aristotle's answer is that the ingredients neither persist *actually* nor are they destroyed (327b23-32).

[2] But compare his remark that "mixture (*mixis*) is not always fusion (*krasis*), for the mixture of dry substances is not fusion" (*Topics* IV.2 122b31-32). Here he did call this *mixis,* so there seems to be an inconsistency in his use of the term *mixis.* Perhaps he was adopting a special technical usage in *De Gen. et Cor.,* for at 328a9 he seems to equate *krasis* and *mixis.* Technical precision may have seemed called for. Plato had used several different terms for mixture fairly loosely in just a few lines of the *Sophist* (252e-253b), and in the *Laws* he had used *symmixis* to mean sexual intercourse (VIII 839a).

I find this idea of a quantity of *matter* existing potentially-but-not-actually very mysterious and in conflict with Aristotle's general views on matter and potentiality. I will discuss this in the last section of this paper. In any case, Aristotle *seems* to believe that an ingredient's persisting-but-not-actually is logically related to its not being-preserved-in-small-particles. For he demands

Axiom 3. A mixture must be *homeomerous*, that is, made up of like parts which are like the whole, just as any part (*meros*) of water is water (328a11-12).

2. *Homeomerism.*[3] What is it to be homeomerous? Leonard and Goodman[4] define the related property of *being dissective* as a property of properties or predicates: a predicate **P** is *dissective* if and only if, if x is part of something that satisfies P, then x will satisfy P:

$$\mathbf{P} \text{ is } \textit{dissective} \text{ iff } x \leq (\text{ə}y \cdot \mathbf{P}y) \supset \mathbf{P}x^{5}$$

For example, the property of being stuff in my pocket is dissective. However, being homeomerous is not the same thing as being dissective, for it is *properties* that are dissective or not, whereas Aristotle seems to call *kinds* of stuff and *quantities* of stuff homeomerous; his own example was water. Being stuff in my pocket may be a dissective property, but I don't think that *the* stuff in my pocket should be called homeomerous.

Aristotle's own illustration refers to the *kind* of stuff water itself: he says ". . . just as any part of water is water." And in Ch. 1 he had characterized the homeomeries of Anaxagoras as things *synonymous with* (named the same as) their parts (314a19-21). So let us try

D1. A kind K^* is *homeomerous* iff $x \leq K^* \supset \mathrm{K}x$.

[3] Parts of sections 2-5 are technical, and parts are digressions. If the reader insists on skipping them or reading them last, he should at least read the bit on "Zeno's Blender" at the end of this section. That shows the basis of my view that two quantities are homeomerously mixed if and only if they occupy the same space at the same time.

[4] "The Calculus of Individuals and Its Uses," *Journal of Symbolic Logic*, 5 (1940) 45-55, at 55.

[5] Remark on the notation: I will use '$\leq$' for the *being part of* relation, and the schwa 'ə' to form an indefinite description 'a/sm P'. Helen Cartwright, in "Heraclitus and the Bath Water," *Philosophical Review*, 74 (1965) 466-485, at 469-472, has offered the exciting suggestion that 'some' with its pronunciation "sm" is the English indefinite article for mass and plural terms. I will occasionally spell 'some' this way to call attention to that usage.

'S(ə$x \cdot$ Px)' is analyzed as '$(\exists x)(\mathrm{P}x \cdot \mathrm{S}x)$'. I also assume that variables are universally quantified unless otherwise indicated.

(The '*' is to remind us that this is '*K*' as a variable ranging over kinds, not a predicate as in 'K*x*'.) There are some difficulties in making sense of this, however. First, to be part of a *kind* is not what it might seem. The water now in my glass is not a part of water; the shrimp salad now on my plate is not a part of shrimp salad. What are some parts of shrimp salad then? Well, one part of shrimp salad is shrimp.

Only kinds are parts of kinds. A performance of Beethoven's *Seventh Symphony* is not a part of it; the viola part, itself an abstract entity, is a part of it. "Thus the semicircles will be parts not of the universal circle but of the particular circles" (*Metaph.*, VII.11, 1037a3–4; see also 1035b). The defining clause in D1 would be satisfied by any elementary kind, since an elementary kind would have no parts (in this sense) but itself. But no elementary kind is a mixture, and we were looking for an analysis of being homeomerous that might apply to mixtures. Aristotle clearly faults Anaxagoras for this error (*De Caelo* III.4, 302b12–20), i.e. for the error of confusing being an element (*stoikheon*) with being homeomerous.

Our difficulty was with the '$x \leq K^*$' clause in D1. 'K^*' is presumed here to have *mass terms* as substituends. Now 'K^*' occurs in a noun position, e.g., as a variable, in that clause. It is useful here to apply Quine's view that a mass term in a substantive position is the proper name of a single concrete object (usually large and scattered): the totality of all that satisfies K.[6] So if we read Aristotle's 'any part of water' as 'any part of the world's water', D1 becomes

D2. A kind K^* is *homeomerous* iff $x \leq$ the world's K $\supset$ Kx.

Unfortunately, on this definition it will turn out that for any mass kind K^*, that kind K^* will be homeomerous if and only if the property of being sm K is dissective. That is, D2 just collapses being homeomerous and being dissective. To show this, we need to show that

(1) $x \leq (\text{ə}y \cdot \text{K}y)$

and

(2) $x \leq$ the world's K

imply each other when K is a mass predicate.

An analysis of definite descriptions built from mass predicates is needed here, since 'the world's K' is such a definite description. I

[6] *Word and Object* (Cambridge, Mass.: M.I.T. Press) p. 98.

have shown elsewhere that the Russellian analysis will not do for a definite mass description such as 'the coffee in my cup', since there is not a unique object satisfying the predicate 'is sm coffee in my cup'.[7] The coffee in the northern half of the cup satisfies it, and so does the coffee in the southern half. What the definite mass description 'the coffee in my cup' designates is the total sum or "fusion" of the various quantities of coffee in my cup—the total quantity of coffee in my cup. If a mass predicate has any instances, the definite mass description formed from it must designate something, and what it designates must itself satisfy that predicate: it must be the case that the coffee in my cup is sm coffee in my cup. This is supported by Quine's observation that mass terms are *cumulative*: "any sum of parts which are water is water" (p. 91).

My notation for 'the G' is '$(\theta x \cdot Gx)$'. (Sharvy, 1980, used a *mu*; I now prefer a *theta*.) My analysis of this is '$(\exists x)(Gx \cdot (y)(Gy \supset y \leq x))$' —sm G that all G is part of. 'F(the G)' is then written '$(\theta x \cdot Gx)Fx$', and the analysis of this is

$$(\exists x)(Gx \cdot (y)(Gy \supset y \leq x) \cdot Fx)$$

This differs from the Russellian analysis only in having the *part of* relation where Russell had identity. Since the world's K is itself sm K, (2) implies (1), and so any dissective mass predicate would be homeomerous on definition D2. Conversely, (1) implies (2) for every x also. For the mass predicate K must be cumulative (Quine's condition), and if $(\exists y \cdot Ky)$ exists and K is cumulative, then $(\theta y \cdot Ky)$ exists also. (That is, if sm K exists and K is cumulative, then the world's K exists also.) Furthermore, $\leq$ is transitive, and $Kx \supset x \leq (\theta y \cdot Ky)$. (That is, any K must be part of the world's K.) So if (1), that is, if x is part of sm K, say of a, then since Ka, a is part of the K, and then so is x. Therefore (1) implies (2) for every x.

So D2 does not analyze Aristotle's notion of homeomerism. On D2, being homeomerous just amounts to being dissective. Recall why being dissective did not work: the property of being stuff in my pocket was dissective, regardless of what was in my pocket. So on D2, the mass kind *stuff in my pocket* would be a homeomerous kind. On the other hand, perhaps whisky-and-water is the sort of thing that should be called homeomerous, yet it fails to satisfy the defining clause in D2: the water in my whisky-and-water is *part of* the world's whisky-and-water, but it is not whisky-and-water.

[7] "A More General Theory of Definite Descriptions," *Philosophical Review*, 89 (1980) 607-624.

Now it might seem that a definition of homeomerism could be given along the above lines by restricting the application of 'part' to *spatially determined* parts. Being whisky-and-water is not a dissective property, for the reasons just given. But what seems to be important is that every *spatially determined* part of a quantity of well-mixed whisky-and-water is sm whisky-and-water. A spatially determined part is a part which is the complete contents of a quantity of space, and if the whisky and the water are well-mixed, it would seem that the contents of any region within the stuff would contain both whisky and water. However, this will not work as a definition of homeomerous, because, again, the property of being stuff in my pocket would turn out to be homeomerous; and that is quite counter-intuitive.

A restriction to so-called "natural kinds" is of no help here either. Water is a natural kind, unlike whisky-and-water which is an artificial kind. But like whisky-and-water, water is a combination of two more elementary kinds. Water should turn out to be homeomerous, just like whisky-and-water, but the property of being water is not dissective: the hydrogen in the world's water is not water.[8]

Aristotle has more to say about his example of wheat and barley particles which fail to be a true mixture. Such a combination fails to be a true mixture because the ingredients continue to exist unaltered; this definitely does involve certain *spatial* notions. Aristotle notes that we might divide the particles into smaller particles, which might yield an apparent mixture, depending on the observer's sharpness of vision. But this would still not be a true mixture "to the eyes of Lynkeus" (328a15-16).

Aristotle also observes that it is *impossible* to divide the wheat and barley particles and arrange them so that every quantity of wheat is *next to* a quantity of barley (328a13-18). So long as there are particles of wheat, these will have spatially determined proper parts which are also wheat; thus, there will be quantitites of wheat completely internal to each wheat particle, which will therefore not be next to any quantity of barley (328a1-5). We can imagine a machine called *Zeno's Blender* that splits particles in two and then

[8] For more on this, see my papers "The Indeterminacy of Mass Predication," in F. J. Pelletier, ed., *Mass Terms: Some Philosophical Problems* (Dordrecht: D. Reidel, Synthese Language Library, 1979) 47-54; "Réponse à Monsieur Pelletier" (forthcoming in *Logique et Analyse*); and "Mixtures," *Philosophy and Phenomenological Research*, 43 (1982-83) at press.

shakes them up and re-arranges them. Suppose that we begin with an arrangement like this:

W	B
B	W

Then, we run it through Zeno's Blender a thousand times. The result will still contain the same figure, and within each wheat particle there is stuff that is only wheat, and not a mixture of wheat and barley.

We need to approach the analysis of homeomerism with notions of spatial division. Homeomerism is a *topo*logical notion, not a purely *mereo*logical one. Although reading *meros* (part) in Axiom 3 as '*spatially determined* part' did not quite work, it did give us a hint in the right direction; it did work for whisky-and-water. What we need is a definition that applies to a quantity thought of as a sum of discrete parts. And we need a spatial or topological component in our definition.

3. *Some Mereology and Topology.* A useful notion is a *partition* of a quantity of stuff:

D3. A set S of subquantities of a quantity Q is a *partition* of Q if and only if (i) no two members of S overlap, and (ii) the sum of S is Q.

Overlapping here is the strict notion from the calculus of individuals.[9] Two quantities overlap just if they have a common part. (I generally prefer to use *being part of* as the primary relation, with overlapping as defined.)

It is vital here not to assume (as Goodman does tacitly) that overlapping is the same as *spatial* overlapping. For example, the light in my room does not overlap the air in my room, although *the spaces they occupy* do overlap. It is easiest to see this point if we think of overlapping defined as having a common part: the air and the light are discrete. An example of a partition would be {the whisky, the water}, which partitions the whisky-and-water. (Please just

[9] See Nelson Goodman, *The Structure of Appearance*, 3rd. ed., Boston Studies in the Philosophy of Science, vol. 53 (Dordrecht: D. Reidel, Synthese Library, 1977).

forget that whisky itself contains water, or make up an example of your own.)

Call the quantity of space occupied by something its *receptacle*. I take this term from Richard Cartwright's "Scattered Objects"[10] although I have changed the sense slightly. (Specifically, Cartwright's receptacles are sets of points of space, and I do not wish to presuppose that a quantity of space is a set of points.) Then we can approach Aristotle's discussion of the wheat and barley problem by first defining relative homogeneity:

> D4. A partition S of a quantity Q is *d-homogenous* if and only if every spherical region of space within the receptacle $r(Q)$ of Q having diameter greater than d overlaps the receptacle of each member of S.

For example, {the beans, the rice} is a one centimeter homogenous partition of my beans and rice, but it is not a one millimeter homogenous partition of it. Or consider a white and black chessboard with two-centimeter squares. {the white, the black} is a two centimeter homogenous partition of the color, but not a 1.9 centimeter homogenous partition of the color. (Treat 'sphere' appropriately, i.e., here as a two-dimensional sphere, i.e., a circle.) Imagine bisecting each square horizontally and vertically and recoloring the board. Then {the white, the black} will be one centimeter homogenous, but not less.[11] Then Aristotle's remarks about the wheat and the barley can be put this way: no matter how many times the wheat and barley particles are put through Zeno's Blender, it will never happen that {the wheat, the barley} is a *zero-homogenous* partition of the wheat and barley.

This suggests a definition of homeomerous:

> D5. A partition S of a quantity Q is *homeomerous* if and only if S is a zero-homogenous partition of Q.

At least this seems to be just what it is *not* possible to reach by repeatedly running the wheat and barley through Zeno's blender. However, it is the limit thus approached.

Notice that being solid rather than liquid is not a determining factor. Olive oil does not *mix* with vinegar. If you put some of each

[10] In Keith Lehrer, ed., *Analysis and Metaphysics* (Dordrecht: D. Reidel, 1975) 153–171, at 153.

[11] There are some problems with this definition that I first noticed during a course of lectures I gave on mixtures in Auckland in 1980. We were not able to solve them. Exercise: figure out what the problems are and then solve them.

in a bottle, even after some shaking, the oil and the vinegar are unmixed. They might form a one millimeter homogenous partition of the total, but the oil is preserved in "particles," i.e., in receptacles containing only oil.

I have avoided any requirement that the partition of a quantity be one that divides it up into "natural kinds." This would be an obvious condition to add, but it would leave us with a definition as soft as the notion of a natural kind itself. If I have a glass of water and you have one too, if we pour them into a single container, the partition {your water, my water} will fail to be homeomerous for the first few minutes, until the stuff is thoroughly mixed.[12] That partition will continue to partition the sum quantity of water even after they are mixed, although it is certainly not a partition into natural kinds. We can easily define a notion of "natural" homeomerism in the manner of D5, simply by adding the requirement that the partition S partition the quantity Q into natural kinds. That notion is then just as clear or unclear as the notion of natural kind, but the softeness is not due to any problem in the underlying definition D5.

Notice that if, in the above example, your water was at 10°C and mine was at 90°C, the sum of your water and my water does not have a defined temperature until they are mixed and reach thermodynamic equilibrium; it is just false that temperature is defined as *average* kinetic energy of molecules for just any old quantity, such as your water + my water *before* they are mixed.

But if a homeomerous mixture, as defined in D5, cannot be created by splitting particles, how is any homeomerous partition possible? If the water in my whisky-and-water does *not* exist as tiny particles, how does it exist? Aristotle attempts to answer these questions beginning at 328a18. But before turning to his answers, I want to discuss some further aspects of homeomerism.

What is the receptacle occupied by the whisky in my whisky-and-water? If $\{a, b\}$ *were* a homeomerous partition of a quantity Q, what would be the receptacles of a, b, and Q? The answer is that all would have the same receptacle. If a is homeomerously a proper part of Q, still, the *content* $c(r(a))$ of the receptacle of a is identical

[12] Perhaps Aristotle would not count this as a mixture, since the quantities would not seem to be separable again. Questions: can the notion of a natural kind be connected with the requirement of separability? Do quantities belong to two natural kinds just if they can be separated if mixed? What sort of possibility is intended in the requirement that they "can" be separated again? See also Denis Robinson, "Re-Identifying Matter," *Philosophical Review*, 91 (1982) 317-341.

with the quantity Q itself. I shall suggest two models on which something might be homeomerous.

4. *The Density Model.* Let the points within my glass which are a rational number of centimeters from its center be called *whisky points,* and the rest *water points.* Then {the whisky points, the water points} is a partition of the *set of points* within the glass, in the usual set-theoretic sense of 'partition'. And every sphere within the glass contains both whisky points and water points. Furthermore, the mixture of points is not merely relative to sharpness of vision—it is logically impossible to have eyes sharp enough to discriminate between the whisky points and the water points. The whisky points are not *next to* any water points, for no point is next to any point at all (*Phys.,* VI.1, 231b7-10). However, each whisky point is arbitrarily close to some water point.

But this density model is not very satisfactory. First of all, it is just absurd to think that actual quantities of whisky and water could be mixed this way. (*De Gen. et Cor.,* I.5, 320b2-4: matter cannot occupy just a point; b16: matter has points and lines as limits.) More crucially, it used a partition of a *set of points* rather than of a *quantity of stuff.* This conflicts with Aristotle's theory of space.

Aristotle holds that quantities of space are not collections of points (*Phys.,* VI.1-2). Nor is any point *part* of any quantity of space; rather, it is quantities of space that are parts of quantities of space; it is lines that are parts of lines (cp. *Phys.,* IV.11, 220a20-23). In Aristotle's topology, *quantities of space* are the most primary objects, and then planes and then lines and then points are defined in terms of them, as limits. Specifically, a *limit* of something is the first point beyond which no part of that thing can be found, and the first point within which every part is contained (*Metaph.,* V.17). This "least upper bound" definition nicely leaves it open whether a limit of something is contained in it or not. (It is interesting to compare mathematical notions of limit that came twenty-two centuries later.)

Aristotle's topology is thus methodologically like Tarski's 1927 geometrical model of mereology, in which spheres and the inclusion relation are primitive, and points are then constructions from these.[13] Tarski notes that a geometric model of mereology (the so-called calculus of individuals) occurs by interpreting "the relation

[13] See his 1927 paper, "Foundations of the Geometry of Solids," in his *Logic, Semantics, Metamathematics* (Oxford: Clarendon Press, 1957) 24-29.

of a part to a whole as the [set] inclusion relation restricted to non-empty regular open sets" (p. 29).

Richard Cartwright employs a number of topological notions and suggests that a set of points of space is a receptacle (i.e., one with which a material object could coincide) if and only if it is a non-empty regular open set, i.e., a non-empty set of points of space identical with the interior of its closure, i.e., identical with the result of removing the limit points from the result of adding all the limit points.[14]

In addition to such structures as the regular open algebra of a topological space, there are also such things as regular *closed* algebras of such spaces. Each exhibits the same structure; intuitively, the difference amounts to whether a thing's border is in it or not. This strikes me as a real "don't care"—if the Oregon-Washington border is part of each state, then those states have points in common, which seems odd; if it is part of neither, then a road crossing from one to the other has points that are in neither, which also seems odd; and it would be odd for that border to belong to one of these states rather than the other. But if we simply think of the mereology of receptacles as primitive, and stick with Aristotle's view of points as defined as limits, these little puzzles do not arise.

Now Aristotle's claim that no combination of quantities preserved in particles is homeomerous implies this:

Theorem: If S partitions Q, and if the set of receptacles of members of S partitions the receptacle $r(Q)$ of Q, then S is not a homeomerous partition of Q.

This is so regardless of whether quantities of space are thought of as non-empty regular open sets, non-empty regular closed sets, or as primary undefined objects.

Aristotle saw a contradiction which seems to escape writers of chemistry textbooks today. On its first page, a chemistry textbook will tell the reader about the atomic theory of matter: all matter consists of tiny things called atoms, etc. Then the reader is treated to distinctions between compounds, solutions, mixtures, suspensions, colloids, alloys, etc. (with definitions that differ from one text to the next). But the typical definition of a solution of x requires that x be *uniformly distributed* (!) in a *continuous medium* (!). This is just what Aristotle saw was impossible if matter comes

[14] Cartwright, *op. cit.*, pp. 155-157. For more on regular open algebras, see Paul Halmos, *Lectures on Boolean Algebras*, §4 (Princeton: Van Nostrand, 1963).

in particles. But he was not an atomist, and so he could go on and still discuss mixtures.

Now suppose that $\{a, b\}$ is a homeomerous partition of a quantity of stuff Q. Then *every* sphere within the receptacle $r(Q)$ of Q overlaps both $r(a)$ and $r(b)$, by the definition of being homeomerous, since every sphere has diameter greater than zero. Now every quantity of space can be thought of as a sum of spheres (they are a topological basis), so $r(a)$ and $r(b)$ are overlapped by *every* quantity of space within $r(Q)$. Hence, $r(a) = r(b) = r(Q)$. That is,

> Theorem: S is a homeomerous partition of Q if and only if every member of S occupies the same space, which is also the space occupied by Q.

So, we have homeomerous quantities just if we have quantities that have no common part, yet which occupy the same space.

This fits in with the density model. If we think of the space occupied by a set of points as its topological closure (or the interior thereof), then we can have completely disjoint sets of points which occupy the same space; this will happen if the two sets are dense in each other in the manner of the whisky-points and the water-points discussed above. {The whisky, the water} is a homeomerous partition of my whisky-and-water just if the whisky itself occupies the same space as the water and as the whisky-and-water.

It is not clear to me that Aristotle saw this implication of his notion of homeomerism, although he does say in the *Physics* that if a quantity is both continuous and homeomerous, its parts have only *potential* receptacles (places); but when the parts are divided but in contact, as in a heap (beans and rice?), then they are actually in places (IV.5, 212b3-6). And Thomas comments "if the elements in a mixture are in distinct positions we only have an apparent mixture."[15]

Aristotle does discuss the idea of two bodies (*sōmata*) being in the same place, and definitely implies that this is impossible (*Phys.*, IV.1, 209a6, 6, 213b18-20, 7, 214b7, *De Gen. et Cor.*, I.5, 321a8-10). But this is always stated in terms of *bodies*. Could two quantities of matter be in the same place? At 320b8-10 he says that if air comes to be from water, it is because its matter is in the water as in a container, and that nothing prevents an infinite number of matters being contained in the water. And what is needed for a mixture is not two *bodies* in the same place, but two quantities of stuff.

[15] Thomas Aquinas, *Summa Theologica,* Part I, Bk. VI, Q. 76, Reply Obj. 4.

But I see no reason to worry about this. Since Schrödinger and De Broglie, we have been encouraged to think of gold, iron and wood as not so different from light, heat and sound.[16] A quantity of sound can occupy the same space as a quantity of air. Finally, notice that chemistry itself routinely speaks of the volume of the nitrogen gas in my room as being equal to the volume of the oxygen and equal to the volume of the sum of these two. Otherwise, the gas laws relating pressure and volume make no sense, especially for partial pressures. And a wine bottle's claim to contain 12% alcohol *by volume* is nonsense—the receptacle of the alcohol is that of the wine itself. Probably what is meant is a conditional: if the alcohol were separated, then its volume would be 12% of the total. But recall that mixing 10cc. of water with 10cc. of alcohol yields about 19cc. of mixture. *While it is still mixed,* the volume of the alcohol is 100% of the volume of the wine.[17]

If a geometric model is wanted which doesn't have the faults of the density model, we can use

5. *A Projection Model.* On this, a quantity of matter is four-dimensional, and the space it occupies is its three-dimensional spatial projection, like a shadow. Notice that his allows discrete quantities to have the same receptacle. In fact the only formal requirement on a receptacle function is this:

> for any set S of quantities, the receptacle $r(\mathrm{Fu}(S))$ of the fusion of S is the fusion $\mathrm{Fu}\{x\colon x = r(\ni y \cdot y \in S)\}$ of the receptacles of the members of S.

This holds both in the density model (on which the receptacle of a set of points would be the interior of its closure) and in the projection model. From this, other equalities and inequalities will follow, such as

$$x \leq y \supset r(x) \leq r(y)$$
$$r(x \cap y) \leq r(x) \cap r(y).$$

[16] See E. Schrödinger, *Science and Humanism,* (New York: Cambridge, 1951), and "Conceptual Models in Physics," in his *Science and Human Temperament* (New York: Norton, 1935); Louis De Broglie, *Matter and Light* (New York: Dover, 1937); and N. R. Hanson, "The Dematerialization of Matter," in E. McMullin, ed., *The Concept of Matter* (Notre Dame, Ind.: University Press, 1963). (This book is out of print. Many of the articles in it appeared later in two other collections of articles on the history of the concept of matter edited by McMullin, but the Hanson article was not among them.)

[17] Discussions of matter and density can found in Bernard Bolzano, *Paradoxes of the Infinite,* §§50-69 (London: Routledge & Kegan Paul, 1950); David Sanford, "Volume and Solidity," *Australasian Journal of Philosophy,* 45 (1967) 329-340; and E. Schrödinger, *op. cit.,* esp. pp. 11-47. See also Robinson.

In fact mathematicians call any such function r on a distributive lattice a *projection*.[18] Notice that the function $m(x)$, *the matter of* x, is itself a projection. If a = the set of rabbits, and b = the set of integral proper rabbit parts, then $a \cap b$ is empty, and so is $m(a \cap b)$. But $m(a) = m(b)$ = the world's rabbit stuff. This projection model was suggested by the literal notion of dimension, but has wider application. The notion of topological closure used in the preceding section to picture the receptacles of sets of points also satisfies the above requirements.

Questions about the dimension of something can become very complicated for exotic mathematical structures. The intuitive notion was nicely described by Poincaré in 1912:

> . . . lines, which can be divided by cuts which are not continua, will be continua of one dimension. . . . If to divide a continuum C, cuts which form one or several continua of one dimension suffice, we shall say that C is a continuum of *two dimensions*; if cuts which form one or several continua of at most two dimensions suffice, we shall say that C is a continuum of *three dimensions*; and so on.[19]

Roughly, the idea is that the dimensional number assigned to something is one higher than the number assigned to the sorts of objects that form its boundaries, the things which separate it from other objects. I will leave for another place the investigation of the relations between homeomerism and dimension.

The projection model was based on the literal idea of geometrical dimension, but it tells us nothing about *how* it can happen that two quantities of matter could occupy the same three-space, yet differ at a fourth co-ordinate and so be disjoint. However, it is one way in which we can have a homeomerous mixture of disjoint quantities of stuff.

Recall Plato's famous remarks on the projection model (*Republic* VII, 515b–c) and the underdetermination of theories by observations. His natives are watching shadows in a cave, projections, and carrying on quite sensible conversations, at least when they stick to observation sentences. But they also think that they are making *references*. According to Plato, it is just here that their talk should be viewed as theoretical rather than purely observational; it is here, says Plato, that the problems arise. Quine's solution is that

[18] Proofs: $x \leq y$ iff $x \cup y = y$, so $r(x \cup y) = r(x) \cup r(y) = r(y)$, so $r(x) \leq r(y)$. Now $x \cap y \leq x$, so $r(x \cap y) \leq r(x)$, and similarly $r(x \cap y) \leq r(y)$; then $r(x \cap y) \leq r(x) \cap r(y)$. See also Halmos, §7.

[19] Quoted in W. Hurewicz and H. Wallman, *Dimension Theory* (New York: Oxford, 1941, and Princeton, N.J.: University Press, 1948), p. 3.

nothing can be said; Plato thinks that something can be said—that these natives are just mistaken. "If they could discuss things with each other, don't you think they would maintain that they were referring to the things that they see passing before them?" (515b). Plato, that is, thinks that we can, from outside the story, determine what they are "really" referring to. In Plato's description, the real things were simply objects of greater spatial dimension. (Notice that shadows, like ingredients in a mixture, can pass through each other.)

In any case, the projection model is purely geometric, and although it gives us a mathematical representation of a homeomerous mixture, it does not really explain how such things might really work. Art is representation; science is representation that explains. The projection model falls short here. It is very, very suggestive, and brings together a number of philosophical questions under a single picture. We have two projections which are very similar: the matter of a thing, and the receptacle of sm matter. Just as two things (such as I and my body) or two sets of things (such as the set of rabbits and the set of integral rabbit parts) might have the same matter, two quantities of stuff, such as the whisky and the water, might have the same receptacle. Although this picture of things falls short of being a scientific explanation, it is a suggestive picture with many applications.

6. *Potentiality and Matter.* Aristotle turns to a discussion of action and receipt of action. Agents which do not have the same matter as their patients act but are not acted upon, and thus cannot mix: the art of healing does not mix with the bodies of patients (328a21-23). This suggests another axiom about mixtures:

> Axiom 4. Two things can mix only if they can act reciprocally on each other, and this requires that their matter be similar.

Not every such combination will be a mixture, because the ingredients must be within an appropriate range of proportion, or one may be destroyed, in violation of a previous principle. A drop of wine added to ten thousand measures of water does not mix; the form of wine is lost; the wine becomes part of (!) the resulting total quantity of water (328a27-28). So

> Axiom 5. There must be a balance between the active powers of the various ingredients.

Now Aristotle had already briefly indicated his explanation of how ingredients could persist in a mixture—they persist potentially.

Suppose that I drop some ice cubes into a glass of good strong tea. I do not immediately have any more tea, but as the ice melts, it becomes part of what then has become tea. Here, it seems that homeomerism is involved in the difference. Some quantities which were merely *in* something become *part* of it when they merge homeomerously with it. In the film *The Invisible Man,* the title character puts on a dressing gown when he has a visitor just after eating dinner. The point was to spare his visitor the sight of chewed food floating about the room at stomach height. The invisible man explains that food he eats becomes invisible only when it is digested and thus becomes part of him. (Other possible questions are tastefully ignored.) Similarly, water preserved in particles, as in a cloud, is not *part* of the air, but is merely *in* the air like a bird or an airplane. But if it evaporates, it ceases to exist in particles, and be comes part of the air. Aristotle frequently speaks of water *coming to be from* air, and of air *coming to be* from water (e.g., *Phys.,* IV.7, 214b3, VII.4, 255b18-19, *De Gen. et Cor.,* I.2, 317a28-30); he also speaks of water being potentially air, and air potentially water (*Phys.,* IV.5, 213a4-11).

Question: do we have in examples of homeomerism examples of a quantity of matter existing potentially? I believe that this is not coherent. The notion of potential existence has application to an individual substance: a particular bust of Hermes may have existed for only 20 years, although its matter, the wood in it, has existed for much longer (cp. *Metaph.,* IX.6, 1048a31-33), and the Hermes had only a potential existence in the original block of wood. But this explanation requires that an individual substance be something other than its matter. It requires that being a bust of Hermes is not a property of that matter. Rather, it is *constituting* a bust of Hermes that is a property of the matter. For if being a bust of Hermes were a property of that matter, then the singular term 'the bust of Hermes' would in fact designate the wood rather than the bust—or would ambiguously designate both.[20]

In that passage in the *Metaphysics,* we have the Hermes described as existing potentially inside the block of wood. This *requires* that the Hermes be distinct from its matter—otherwise, it would come to exist in a *de dicto* sense only. That *de dicto* sense is the one in which if my coffee becomes cold, then some cold coffee

[20] This is an instance of a general principle that is often missed. Here are some other instances of the principle: if being air were a property of the space in this room, then 'the air' would designate the space; if 'is a skillet' were true of the iron in my skillet, and 'is iron' were true of the space it occupied, then it would be true that the skillet = the iron = the space.

has come into existence. But the sugar in the coffee cannot be like the Hermes in the wood. Whereas the Hermes comes to exist *de re* when the sculptor carves the wood, the sugar is *matter*, and must exist throughout the process of dissolving and separation.

The notions of form and matter are used to explain the becoming and destruction of individual substances. But the matter involved is something which itself is not subject to becoming or destruction. So what sense does it make to say e.g. that the beans in my beans and rice exist actually, but that the sugar (dissolved) in my coffee exists only potentially? This is a mistake, since the sugar is itself sm matter, and cannot have any "further" matter which persists when *it*, that sugar, comes to be or passes away (if it does; but it doesn't). Aristotle says this himself: "For if it came to exist, there must have existed a primary substratum from which it came. . . . For matter is what I call the primary substratum of each thing, from which it comes to exist . . ." (*Phys.*, I.9, 192a28-34).

It would be unfair to ignore another, looser, sense of matter that Aristotle uses, and which might help here. In this looser sense, the matter of something is, in the plainest sense, that of which it consists, is composed, or is made. In this sense, the matter of a deck of cards is the cards, the matter of a syllable is the letters, and the matter of a forest is the trees (*Phys.*, II.3, 195a15-19). On this account, the trees in turn have matter—the wood of which they are composed, and the wood in turn has matter—prime matter, perhaps. Indeed, this chain yields *one* definition of prime matter: it is the limit of the *matter of* relation.

On this account then, some matter can be destroyed. The trees are the matter of the forest, and they can be destroyed if *their* matter loses its form as trees, perhaps in a forest fire. Is it only prime matter then, that would be indestructible on this account? Notice that the sugar is matter in a sense in which the trees are not. It is matter in the unqualified sense, i.e., the non-relative sense. Although the trees are matter only in that they are the matter *of* something, the sugar is matter even if it is not the matter of anything. It need not exist as a lump; it can just be a formless heap (cf. *Metaph.*, VIII.16-17, VIII.6).

The sugar that I added to my coffee may be homeomerously in the sugar + the coffee, but it certainly exists. *It might not still be sugar*—but *it*, that stuff, cannot pass away. It may also be potentially visible sugar, and actually tasteable potential sugar—it can be separated again by evaporation of the water. But *it*, whether actually sugar or not, still exists. Matter is ultimately the substratum of coming to be and passing away. That is, no matter itself is ever created or destroyed. Aristotle's remark at *Metaph.*, IX.8, 1050a15-17, that matter exists potentially just because it *may* attain form, just seems crazy. For what is the it? What is the it, unless something that actually exists?? Where is the snow of yesteryear? All around us, and in us.

But Aristotle has called our attention to something very interesting:

> There are some kinds whose parts are necessarily homeomerous.

One example is the water in tea. That is exactly the situation we had with the ice cubes melting into a glass of iced tea. The ice cubes are H_2O not homeomerously contained, and so are merely *in* some tea, surrounded by it; when they melt, that H_2O becomes part of some tea. The water droplets in a cloud are merely in the air, but when they evaporate, they become part of the air. So air is a kind for which water is a necessarily homeomerous ingredient. Aristotle seemed to hesitate at saying that there is actually water in air when it is a homeomerous part; I would not hesitate at all, but nothing turns on this difference. What is important is to see that Aristotle *should* have said that whether or not what was water in the cloud is still water when it has evaporated, still, *it* exists as part of the air.

Chemical knowledge may have now changed the actual circumstances under which we learn mass terms. The child learns 'iron' accompanied by mama's skillet, but also in sentences like 'eat your spinach; it's got iron in it'. In any case, what is in the spinach may be potentially iron-in-the-primary-sense, and actually iron-in-a-secondary-sense. So there does not exist in the spinach any $iron_1$. Similarly, when sugar dissolves in water, the $sugar_1$ ceases to exist; yet nothing has actually ceased to exist. Rather, some stuff has changed qualitatively from being $sugar_1$ to being $sugar_2$. This is no more ceasing to exist than the *de dicto* sense in which a cultured man ceases to exist when Ronald, a cultured man, becomes uncultured. (See *De Gen. et Cor.*, I.4, 319b6-31.) We have alteration (rather than generation or destruction) if and only if we have a single matter (I.1, 314b29-315a3).

To the extent that Aristotle's explanation of mixtures depends upon a quantity of matter itself existing potentially at one time and actually at another time, his explanation fails to be consistent with the rest of his theory of matter. The idea of potential existence properly belongs to the form+matter theory of individual substances. But the theory of mixtures is wholly within the pure theory of matter.

On the other hand, he has called our attention to certain kinds of mixtures, and to a relation between the being part of relation and homeomerism. Specifically, there are certain kinds of stuff *A*, quantities of which contain quantities of other kinds of stuff *B* as part if and only if the *B* exists homeomerously in the *A*. An example is the water in the air, which is part of the air if and only if it is in it homeomerously, but otherwise that water is merely in the air like a bird, without being part of the air.

This I find quite exciting.

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FITNESS*

DEBATES about the cognitive status of the Darwinian theory of natural selection should have ended long ago. Their persistence reflects the steady failure of biologists and philosophers of science to treat the notion of fitness as the quite ordinary theoretical term which in fact it is. Even the rare expositions of fitness and its role in evolutionary theory that have been correct have failed to put the methodological controversy over this theory to rest.[1] In this paper I shall show that 'fitness' differs from an ordinary theoretical term, like temperature, not in kind, but only in degree; that this difference sets limits on the measurement of fitness; that these limits give the theory of evolution its undeserved reputation for vacuousness. I then apply these conclusions about fitness to laboratory experiments in evolution, with a surprising re-

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[1] For example, D. Hull, *The Philosophy of Biological Science*, and M. Ruse, *The Philosophy of Biology*, both address the allegation that the theory of natural selection is vacuous but they fail to explain why it persistently attracts this false charge. Other recent attempts to refute the charge, such as Mary Williams, "Falsifiable Predictions of Evolutionary Theory," *Philosophy of Science*, 40 (1973): 518-537, or S. K. Mills and J. H. Beatty, "The Propensity Interpretation of Fitness," *Philosophy of Science*, 46 (1979): 263-288, neither correctly diagnose the source of this error, nor provide effective remedies for it, and they generate some further obstacles to the dissolving of the mistake. For details of these defects see, A. Rosenberg, "The Supervenience of Biological Concepts," *Philosophy of Science*, 45 (1978): 368-386, and "On the Propensity Definition of Fitness," *Philosophy of Science*, 49 (1982): 268-273.

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